Chapter 8 Matrices and Determinants

Course Number
Instructor
Date

Section 8.1 Matrices and Systems of Equations

Objective: In this lesson you learned how to use matrices, Gaussian elimination, and Gauss-Jordan elimination to solve systems of linear equations.

Important Vocabulary	Define each term or concept.
Entry of a matrix	
Order of a matrix	
Square matrix	
Main diagonal	
Row matrix	
Column matrix	
Elementary row operations	
Gauss-Jordan elimination	

I. Matrices (Pages 572–573)

What you should learn How to write matrices and identify their orders

If m and n are positive integers, an $m \times n$ **matrix** is . . .

An $m \times n$ matrix has __________ rows and __________ columns.

An **augmented matrix** is . . .

A **coefficient matrix** is . . .

Example 1: Consider the following system of equations.

$$\begin{cases} 2x + y - z = 5 \\ x - 3y + 2z = 9 \\ 3x + 2y = 1 \end{cases}$$

(a) Write the augmented matrix for this system.
(b) What is the order of the augmented matrix?
(c) Write the coefficient matrix for this system.
(d) What is the order of the coefficient matrix?

II. Elementary Row Operations (Pages 574–576)

What you should learn
How to perform elementary row operations on matrices

The **elementary row operations** on a matrix are:

Two matrices are **row-equivalent** if . . .

A matrix in **row-echelon form** has the following three properties:
1.

2.

3.

A matrix in row-echelon form is in **reduced row-echelon form** if . . .

III. Gaussian Elimination with Back-Substitution
(Pages 577–578)

What you should learn
How to use matrices and Gaussian elimination to solve systems of linear equations

To solve a system of linear equations using Gaussian Elimination with Back-Substitution, . . .

If, during the elimination process, you obtain a row with zeros except for the last entry, you can conclude that the system has ____________________.

Example 2: Solve the following system using Gaussian Elimination with Back-Substitution.

$$\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 1 \\ x - 3y + 5z = -11 \end{cases}$$

IV. Gauss-Jordan Elimination (Pages 579–581)

What you should learn
How to use matrices and Gauss-Jordan elimination to solve systems of linear equations

Example 3: Apply Gauss-Jordan elimination to the following matrix to obtain the unique reduced row-echelon form of the matrix.

$$\left[\begin{array}{ccc:c} 1 & 4 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Example 4: Solve the following system using Gauss-Jordan elimination.

$$\begin{cases} 2x - y + 3z = 1 \\ x + 2y - 4z = -6 \\ -2x + 3y - z = 13 \end{cases}$$

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Exercises 1 – 29 odd

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Section 8.2 Operations with Matrices

Objective: In this lesson you learned how to add and subtract matrices, multiply matrices by scalars, and multiply two matrices.

Important Vocabulary	Define each term or concept.
Scalars	
Scalar multiple	
Zero matrix	
Matrix multiplication	
Identity matrix of order *n*	

I. Equality of Matrices (Page 587)

What you should learn
How to decide whether two matrices are equal

Name three ways that a matrix may be represented.

1)

2)

3)

Two matrices are equal if they have the same order and

____________________________________ are equal.

II. Matrix Addition and Scalar Multiplication (Pages 588–591)

What you should learn
How to add and subtract matrices and multiply matrices by scalars

To add two matrices of the same order, . . .

To multiply a matrix A by a scalar c, . . .

Example 1: Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 2 & -5 \end{bmatrix}$.
Find (a) $A + B$ and (b) $-2B$

Let A, B, and C be $m \times n$ matrices and let c and d be scalars. Give an example of each of the following properties of matrix addition and scalar multiplication:

1) Commutative Property of Matrix Addition: ____________________
2) Associative Property of Matrix Addition: ____________________
3) Associative Property of Scalar Multiplication: ____________________
4) Scalar Identity Property: ____________________
5) Distributive Property (two forms): ____________________

____

If A is an $m \times n$ matrix and O is the $m \times n$ zero matrix consisting entirely of zeros, then $A + O =$ ____________.

The additive identity for the set of all $m \times n$ matrices is the $m \times n$ matrix ____________________.

III. Matrix Multiplication (Pages 592–594)

What you should learn
How to multiply two matrices

When multiplying an $m \times n$ matrix A by an $n \times p$ matrix B, to obtain the entry in the ith row and jth column of AB, . . .

Example 2: If A is a 3×5 matrix and B is a 6×3 matrix, find the order, if possible, of the product (a) AB, and (b) BA.

Example 3: Find the product AB, if

$$A = \begin{bmatrix} 2 & -1 & 7 \\ 0 & 6 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

List four properties of Matrix Multiplication:

If A is an $n \times n$ matrix, the identity matrix I of order n has the property that ______________ and ______________.

IV. Applications of Matrix Operations (Pages 595–596)

> ***What you should learn***
> How to use matrix operations to model and solve real-life problems

Matrix multiplication can be used to represent a system of linear equations. The system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

can be written as the matrix equation __________________, where A is the coefficient matrix of the system and X and B are column matrices.

Example 4: Consider the following system of linear equations.

$$\begin{cases} 2x_1 - x_2 + 3x_3 = -11 \\ x_1 - 3x_3 = -1 \\ -x_1 + 4x_2 + 2x_3 = 2 \end{cases}$$

Write this system as a matrix equation $AX = B$, and then use Gauss-Jordan elimination on the augmented matrix $[A : B]$ to solve for the matrix X.

Additional notes

Homework Assignment

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Exercises 31–65 odd

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Section 8.3 The Inverse of a Square Matrix

Objective: In this lesson you learned how to find the inverses of matrices and use inverse matrices to solve systems of linear equations.

Important Vocabulary Define each term or concept.

Inverse of a matrix

I. The Inverse of a Matrix (Page 602)

What you should learn
How to verify that two matrices are inverses of each other

To verify that a matrix B is the inverse of the matrix A, . . .

II. Finding Inverse Matrices (Pages 603–605)

What you should learn
How to use Gauss-Jordan elimination to find the inverses of matrices

If a matrix A has an inverse, A is called ________________ or nonsingular. Otherwise, A is called ________________.

To have an inverse, a matrix must be ______________. Not all square matrices have inverses. However, if a matrix does have an inverse, that inverse is ______________.

To find the inverse of a square matrix A of order n, . . .

Example 1: Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 6 \end{bmatrix}$.

III. The Inverse of a 2 × 2 Matrix (Page 606)

> *What you should learn*
> How to use a formula to find the inverses of 2 × 2 matrices

If A is a 2 × 2 matrix given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is invertible if and only if ________________. Moreover, if this condition is true, the inverse of A is given by:

$$A^{-1} = \frac{}{\rule{3cm}{0.4pt}}\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

The denominator is called the ____________________ of the 2 × 2 matrix A.

Example 2: Find the inverse of the matrix $B = \begin{bmatrix} 3 & 9 \\ -2 & -7 \end{bmatrix}$.

IV. Systems of Linear Equations (Page 607)

> *What you should learn*
> How to use inverse matrices to solve systems of linear equations

If A is an invertible matrix, the system of linear equations represented by $AX = B$ has a unique solution given by ________________.

Example 3: Use an inverse matrix to solve (if possible) the system of linear equations:

$$\begin{cases} 12x + 8y = 416 \\ 3x + 5y = 152 \end{cases}$$

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Exercises 67–89 odd

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Section 8.4 The Determinant of a Square Matrix

Objective: In this lesson you learned how to find minors, cofactors, and determinants of square matrices.

Important Vocabulary Define each term or concept.

Determinant

Minors

Cofactors

I. The Determinant of a 2 × 2 Matrix (Pages 611–612)

What you should learn
How to find the determinants of 2 × 2 matrices

The **determinant** of the 2 × 2 matrix $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ is given by

$\det(A) = |A| = \begin{vmatrix} \quad & \quad \end{vmatrix} =$ ____________________

The determinant of a matrix of order 1 × 1 is defined as . . .

Example 1: Find the determinant of the matrix $A = \begin{bmatrix} -4 & 3 \\ 1 & -2 \end{bmatrix}$.

II. Minors and Cofactors (Page 613)

What you should learn
How to find minors and cofactors of square matrices

Complete the sign patterns for cofactors of a 3 × 3 matrix, a 4 × 4 matrix, and a 5 × 5 matrix:

Sign Pattern for Cofactors

3 × 3 matrix 4 × 4 matrix 5 × 5 matrix

$\begin{bmatrix} \quad \end{bmatrix}$ $\begin{bmatrix} \quad \end{bmatrix}$ $\begin{bmatrix} \quad \end{bmatrix}$

Example 2: Use the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ to find:

(a) the minor M_{13}, and (b) the cofactor C_{21}.

III. The Determinant of a Square Matrix (Pages 614–615)

What you should learn
How to find the determinants of square matrices

Applying the definition of the determinant of a square matrix to find a determinant is called ______________________________.

Example 3: Find the determinant of the matrix:

$$A = \begin{bmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Example 4: Describe a strategy for finding the determinant of the following matrix, and then find the determinant of the matrix.

$$B = \begin{bmatrix} -2 & 4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & -1 \\ -5 & 0 & -2 & 3 \end{bmatrix}$$

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Exercises 95– 106

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Section 8.5 Applications of Matrices and Determinants

Objective: In this lesson you learned how to use Cramer's Rule to solve systems of linear equations and how to use determinants and matrices to model and solve problems.

I. Cramer's Rule (Pages 619–621)

What you should learn
How to use Cramer's Rule to solve systems of linear equations

Cramer's Rule states that if a system of n linear equations in n variables has a coefficient matrix A with a nonzero determinant $|A|$, the solution of the system is

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \ldots, x_n = \frac{|A_n|}{|A|}$$

where the ith column of A_i is ______________________________

____.

Cramer's Rule does not apply if the determinant of the coefficient matrix is ______________, in which case the system has either no solution or ______________________________.

Example 1: Use Cramer's Rule to solve the system of linear equations.

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ \quad\ \ y - 2z = -3 \end{cases}$$

II. Area of a Triangle (Page 622)

What you should learn
How to use determinants to find the areas of triangles

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive area.

Example 2: Find the area of a triangle whose vertices are $(-3, 1)$, $(2, 4)$, and $(5, -3)$.

III. Lines in a Plane (Pages 623–624)

What you should learn
How to use a determinant to test for collinear points and find an equation of a line passing through two points

Three points (x_1, y_1), (x_2, y_2), and (x_3, y_3) are collinear (lie on the same line) if and only if

$$\left| \quad \right| = 0.$$

Example 3: Determine whether the points $(-2, 4)$, $(0, 3)$, and $(8, -1)$ are collinear.

An equation of the line passing through the distinct points (x_1, y_1) and (x_2, y_2) is given by

$$\left| \quad \right| = 0.$$

Example 4: Find an equation of the line passing through the points $(-2, 9)$ and $(3, -1)$.

IV. Cryptography (Pages 625–627)

What you should learn
How to use matrices to encode and decode messages

A cryptogram is . . .

To use matrix multiplication to encode and decode messages, . . .

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Exercises 107–119 odd