Chapter 6 Additional Topics in Trigonometry

Course Number
Instructor
Date

Section 6.1 Law of Sines

Objective: In this lesson you learned how to use the Law of Sines to solve oblique triangles and how to find the areas of oblique triangles.

Important Vocabulary	Define each term or concept.
Oblique triangle	

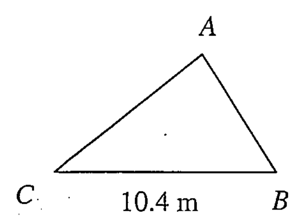
I. Introduction (Pages 430–431)

What you should learn
How to use the Law of Sines to solve oblique triangles (AAS or ASA)

To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle. Describe two cases that can be solved using the Law of Sines.

State the **Law of Sines.**

Example 1: For the triangle shown at the right, $A = 31.6°$, $C = 42.9°$, and $a = 10.4$ meters. Find the length of side c.



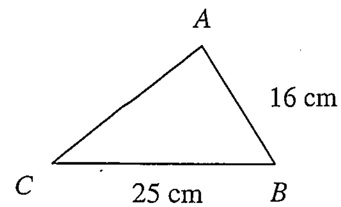
II. The Ambiguous Case (SSA) (Pages 432–433)

What you should learn
How to use the Law of Sines to solve oblique triangles (SSA)

If two sides and one opposite angle of an oblique triangle are given, ________________ possible situations can occur, which are:

Example 2: For a triangle having $A = 25°$, $b = 54$ feet, and $a = 26$ feet, how many solutions are possible?

Example 3: For the triangle shown at the right, $A = 110°$, $c = 16$ centimeters, and $a = 25$ centimeters. Find the length of side b.



III. Area of an Oblique Triangle (Page 434)

What you should learn How to find the areas of oblique triangles

The area of any triangle is ________________ the product of the lengths of two sides times the sine of ______________________.

That is,

Area = ____

Example 4: Find the area of a triangle having two sides of lengths 30 feet and 48 feet and an included angle of 40°.

IV. Applications of the Law of Sines (Page 435)

What you should learn How to use the Law of Sines to model and solve real-life problems

Describe a real-life situation in which the Law of Sines could be used.

Homework Assignment

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Exercises 1–19 odd.

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Section 6.2 Law of Cosines

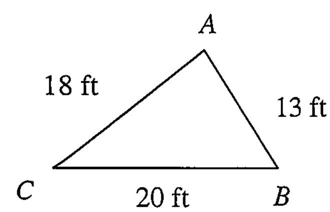
Objective: In this lesson you learned how to use the Law of Cosines to solve oblique triangles and to use Heron's Formula to find the area of a triangle.

I. Introduction (Pages 439–440)

> ***What you should learn***
> How to use the Law of Cosines to solve oblique triangles (SSS or SAS)

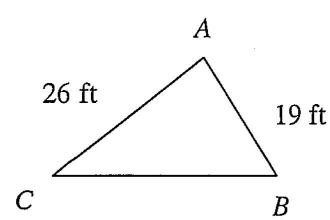
State the **Law of Cosines**.

Example 1: Using the triangle shown at the right, find angle A.



When given the lengths of all three sides of a triangle and asked to find all three angles, which angle should be found first? Why?

Example 2: In the triangle shown at the right, if $A = 62°$, find the length of side a.



II. Applications of the Law of Cosines (Page 441)

> ***What you should learn***
> How to use the Law of Cosines to model and solve real-life problems

Describe a real-life situation in which the Law of Cosines could be used.

III. Heron's Area Formula (Page 442)

> ***What you should learn***
> How to use Heron's Area Formula to find the area of a triangle

Heron's Area Formula states that given any triangle with sides of length a, b, and c, the area of the triangle is:

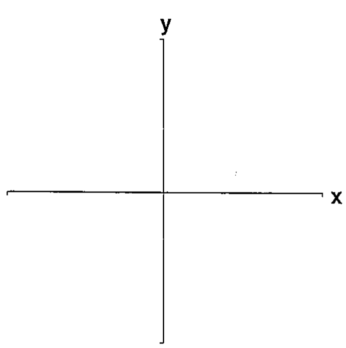
Area = $\sqrt{\rule{8cm}{0.4pt}}$

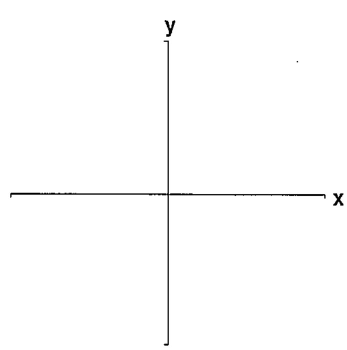
____

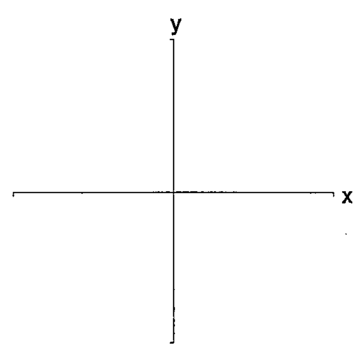
where s = ____________________.

Example 3: Find the area of a triangle having sides of length $a = 14$ cm, $b = 21$ cm, and $c = 27$ cm.

Additional notes







Homework Assignment

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Exercises 21–35 odd

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Section 6.3 Vectors in the Plane

Objective: In this lesson you learned how to write the component forms of vectors, perform basic vector operations, and find the direction angles of vectors.

Important Vocabulary Define each term or concept.

Vector v in the plane

Standard position

Zero vector

Unit vector

Standard unit vectors

Direction angle

I. Introduction (Page 447)

What you should learn
How to represent vectors as directed line segments

A **directed line segment** has an ____________________ and a ____________________.

The **magnitude of the directed line segment** $\overrightarrow{PQ}$, denoted by ______________, is its ______________. The magnitude of a directed line segment can be found by . . .

II. Component Form of a Vector (Page 448)

What you should learn
How to write the component forms of vectors

A vector whose initial point is at the origin (0, 0) can be uniquely represented by the coordinates of its terminal point (v_1, v_2). This is the ____, written $\mathbf{v} = \langle v_1, v_2 \rangle$, where v_1 and v_2 are the __________________ of **v**.

The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

$\overrightarrow{PQ} =$ ____________________ $=$ ____________ $= \mathbf{v}$.

The **magnitude** (or length) **of v** is:

$\|\mathbf{v}\| = \sqrt{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \sqrt{\_\_\_\_\_\_\_\_}$

Example 1: Find the component form and magnitude of the vector **v** that has (1, 7) as its initial point and (4, 3) as its terminal point.

III. Vector Operations (Pages 449–451)

> ***What you should learn***
> How to perform basic vector operations and represent them graphically

In operations with vectors, numbers are usually referred to as ______________. Geometrically, the product of a vector **v** and a scalar k is . . .

If k is positive, $k\mathbf{v}$ has the ______________ direction as **v**, and if k is negative, $k\mathbf{v}$ has the ______________ direction.

To add two vectors geometrically, . . .

This technique is called the __________________________ for vector addition because the vector $\mathbf{u} + \mathbf{v}$, often called the ________________ of vector addition, is . . .

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a scalar (a real number). Then the sum of **u** and **v** is the vector:

$\mathbf{u} + \mathbf{v} =$ ______________________

and the scalar multiple of k times **u** is the vector:

$k\mathbf{u} =$ __________________________

Example 2: Let $\mathbf{u} = \langle 1, 6 \rangle$ and $\mathbf{v} = \langle -4, 2 \rangle$. Find:
(a) $3\mathbf{u}$
(b) $\mathbf{u} + \mathbf{v}$

Let **u**, **v**, and **w** be vectors and c and d be scalars. Complete the following properties of vector addition and scalar multiplication:

1. $\mathbf{u} + \mathbf{v} =$ ______________
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} =$ ______________
3. $\mathbf{u} + \mathbf{0} =$ ____________
4. $\mathbf{u} + (-\mathbf{u}) =$ ____________
5. $c(d\mathbf{u}) =$ ______________
6. $(c + d)\mathbf{u} =$ ______________
7. $c(\mathbf{u} + \mathbf{v}) =$ ______________
8. $1(\mathbf{u}) =$ ____________
9. $0(\mathbf{u}) =$ ____________
10. $\|c\mathbf{v}\| =$ ______________

IV. Unit Vectors (Pages 451–452)

> ***What you should learn***
> How to write vectors as linear combinations of unit vectors

To find a unit vector **u** that has the same direction as a given nonzero vector **v**, . . .

In this case, the vector **u** is called a ______________________ ____________________.

Example 3: Find a unit vector in the direction of $\mathbf{v} = \langle -8, 6\rangle$.

Let $\mathbf{v} = \langle v_1, v_2\rangle$. Then the standard unit vectors can be used to represent **v** as $\mathbf{v} =$ ________________ , where the scalar v_1 is called the ________________________________ and the scalar v_2 is called the ______________________________. The vector sum $v_1\mathbf{i} + v_2\mathbf{j}$ is called a ___________________________ of the vectors **i** and **j**.

Example 4: Let $\mathbf{v} = \langle -5, 3\rangle$. Write **v** as a linear combination of the standard unit vectors **i** and **j**.

Example 5: Let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 9\mathbf{j}$. Find $\mathbf{v} + \mathbf{w}$.

V. Direction Angles (Page 453)

> *What you should learn*
> How to find the direction angles of vectors

If **u** is a unit vector and θ is its direction angle, the terminal point of **u** lies on the unit circle and

$\mathbf{u} = \langle x, y \rangle =$ ________________ = ____________________

Now, if **v** is any vector that makes an angle θ with the positive x-axis, it has the same direction as **u** and

$\mathbf{v} =$ ____________________ = ______________________

If **v** can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, then the direction angle θ for **v** can be determined from $\tan \theta =$ ____________.

Example 6: Let $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$. Find the direction angle for **v**.

VI. Applications of Vectors (Pages 454–455)

> *What you should learn*
> How to use vectors to model and solve real-life problems

Describe several real-life applications of vectors.

Homework Assignment

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Exercises 37– 69 odd

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Section 6.4 Vectors and Dot Products

Objective: In this lesson you learned how to find the dot product of two vectors and find the angle between two vectors.

Important Vocabulary Define each term or concept.

Angle between two nonzero vectors

Orthogonal

I. The Dot Product of Two Vectors (Pages 460–461)

What you should learn
How to find the dot product of two vectors and use the Properties of the Dot Product

The **dot product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is ______________________. This product yields a ___________.

Let **u**, **v**, and **w** be vectors in the plane or in space and let c be a scalar. Complete the following properties of the dot product:

1. $\mathbf{u} \bullet \mathbf{v} =$ _______________
2. $\mathbf{0} \bullet \mathbf{v} =$ ____________
3. $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) =$ ____________________
4. $\mathbf{v} \bullet \mathbf{v} =$ _______________
5. $c(\mathbf{u} \bullet \mathbf{v}) =$ _______________ = _______________

Example 1: Find the dot product: $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$.

II. The Angle Between Two Vectors (Pages 461–463)

What you should learn
How to find the angle between two vectors and how to determine whether two vectors are orthogonal

If θ is the angle between two nonzero vectors **u** and **v**, then θ can be determined from ___________________________________.

Example 2: Find the angle between $\mathbf{v} = \langle 5, -4 \rangle$ and $\mathbf{w} = \langle 9, -2 \rangle$.

An alternative way to calculate the dot product between two vectors **u** and **v**, given the angle θ between them, is

______________________________.

Two vectors **u** and **v** are orthogonal if ______________.

Example 3: Are the vectors $\mathbf{u} = \langle 1, -4 \rangle$ and $\mathbf{v} = \langle 6, 2 \rangle$ orthogonal?

III. Finding Vector Components (Pages 463–465)

> ***What you should learn***
> How to write a vector as the sum of two vector components

Let **u** and **v** be nonzero vectors such that $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$, where $\mathbf{w}_1$ and $\mathbf{w}_2$ are orthogonal and $\mathbf{w}_1$ is parallel to (or a scalar multiple of) **v**. The vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ are called ____________________ ___________. The vector $\mathbf{w}_1$ is the **projection** of **u** onto **v** and is denoted by ___________________. The vector $\mathbf{w}_2$ is given by ___________________.

Let **u** and **v** be nonzero vectors. The projection of **u** onto **v** is given by $\text{proj}_{\mathbf{v}}\,\mathbf{u} =$ ____________________________.

IV. Work (Page 466)

> ***What you should learn***
> How to use vectors to find the work done by a force

The **work** W done by a constant force **F** as its point of application moves along the vector $\overrightarrow{PQ}$ is given by either of the following:

1.

2.

Homework Assignment

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Exercises 73–95 odd

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Section 6.5 Trigonometric Form of a Complex Number

Objective: In this lesson you learned how to multiply and divide complex numbers written in trigonometric form and how to find powers and *n*th roots of complex numbers. .

Important Vocabulary Define each term or concept.

Real axis

Imaginary axis

Absolute value of a complex number $a + bi$

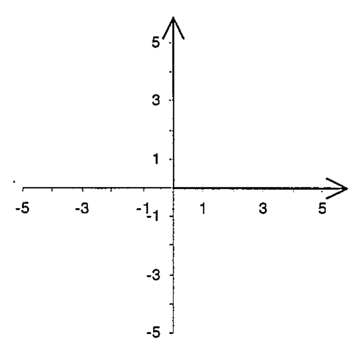
***n*th roots of unity**

I. The Complex Plane (Page 470)

What you should learn
How to plot complex numbers in the complex plane and find absolute values of complex numbers

The **complex plane** is . . .

On the complex plane shown at the right, (a) label the real axis, (b) label the imaginary axis, and (c) plot and label the complex numbers $-2 - 3i$ and $4 + i$.



The absolute value of the complex number $z = a + bi$ is given by

$|a + bi| = \sqrt{}$.

II. Trigonometric Form of a Complex Number (Pages 471–472)

What you should learn
How to write the trigonometric forms of complex numbers

The **trigonometric form** of the complex number $z = a + bi$ is

$z =$ ____________________,

where $a =$ ______________,

$b =$ ______________,

$r = \sqrt{}$, and

$\tan \theta =$ ____________.

The number r is the ______________ of z, and θ is called an ______________ of z.

The trigonometric form of a complex number is also called the

____________________.

III. Multiplication and Division of Complex Numbers (Pages 472–473)

What you should learn
How to multiply and divide complex numbers written in trigonometric form

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ be complex numbers. Then:

$z_1 z_2 =$ ____________________________________

$z_1/z_2 =$ ____

Describe how to find the product of two complex numbers.

Describe how to find the quotient of two complex numbers.

IV. Powers of Complex Numbers (Page 474)

What you should learn
How to use DeMoivre's Theorem to find powers of complex numbers

State DeMoivre's Theorem.

IV. Roots of Complex Numbers (Pages 475–477)

What you should learn
How to find *n*th roots of complex numbers

The complex number $u = a + bi$ is an ***n*th root** of the complex number z if ______________________________.

For a positive integer n, the complex number $z = r(\cos \theta + i \sin \theta)$ has ____ given by $\sqrt[n]{r}\left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n}\right)$, where $k = 0, 1, 2, \ldots, n - 1$.

Homework Assignment

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Exercises 97 – 117 odd