BRIEF PI TALES

80

Various values for the number pi (π) have been used throughout history. In ancient Asia, the value of π was frequently considered to be 3. In the Egyptian *Rhind Papyrus* (1650 B.C.) $\pi = (\frac{4}{3})^4$, or approximately 3.1604. Greek mathematician **Archimedes** (ca 287–212 B.C.) used the fact that the circumference of a circle lies between the perimeter of any inscribed polygon and the perimeter of any circumscribed polygon to determine that π is between $\frac{223}{71}$ and $\frac{22}{7}$. Rounded to two decimal places, his value for π equals 3.14. China's **Zu Chongzhi** (ca A.D. 480) came up with $\frac{355}{113}$, or 3.1415929 . . . , which is accurate to six decimal places. Hindu mathematicians **Aryabhata** (ca A.D. 530) and **Bhaskara** (1114–1185) obtained values of $\frac{62{,}843}{20{,}000}$ and $\frac{754}{240}$, respectively.

These mathematicians are just a few of the many who studied the approximations for π. Some produced amazingly accurate approximations long before the modern age of computers. In 1853, England's William Rutherford computed π accurate to 400 decimal places!

There are various mnemonic devices that can help us remember π to many decimal places. In 1914, the *Scientific American* published this mnemonic statement:

> *See, I have a rhyme assisting my fee-*
> *ble brain, its tasks ofttimes resisting.*

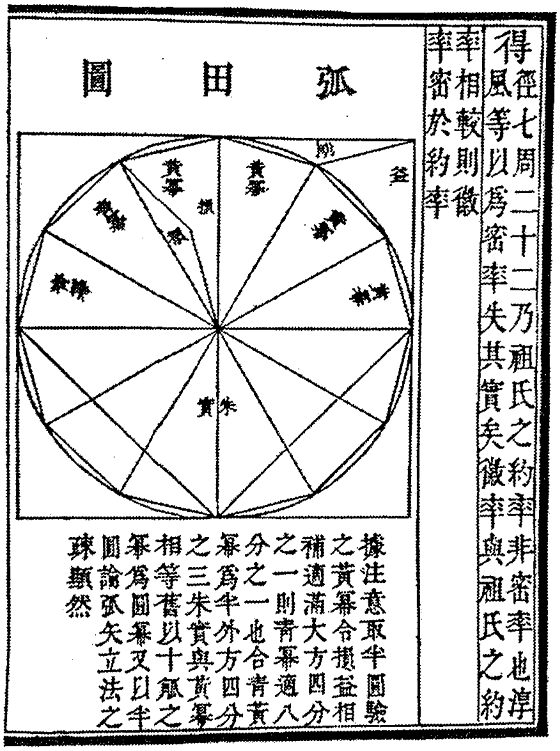
Replacing each word by the number of letters it contains yields π correct to 12 decimal places.

In 1906, A. C. Orr had written a similar but more detailed mnemonic that appeared in the *Literary Digest.* It helps us remember π to 30 decimal places.

> *Now I, even I, would celebrate*
> *In rhymes unapt, the great*
> *Immortal Syracusan, rivaled nevermore,*
> *Who in his wondrous lore,*
> *Passed on before,*
> *Left men his guidance*
> *How to circles mensurate.*

Would you want to memorize that?

So important is the number π that some have actually attempted to legislate its value.



Chinese mathematician Liu Hui's method for finding the approximate value of π (A.D. 264).

In 1897, the Indiana State Legislature came up with House Bill Number 246. Section I of the bill starts like this: "Be it enacted by the General Assembly of the State of Indiana: It has been found that the circular area is to the quadrant of the circumference, as the area of an equilateral rectangle is to the square on one side."

Apparently, the House believed this was an important issue, because they passed the bill with a vote of 67 to 0. However—for reasons that become obvious when you carefully read what is written in the bill—it became the subject of newspaper ridicule and was shelved by the Senate.

Concerning all the ill-fated attempts to supply a specific value for π, mathematics historian Howard Eves wrote: "These contributions, often amusing, and at times almost unbelievable, would require a publication all to themselves." According to him, the authors of the numerous documents claiming to yield an exact value for π suffer from morbus cyclometricus—the circle-squaring disease.

For more on pi, see vignette 2. ★

ACTIVITIES

1. Research the clever method used by Germany's **Ludolph van Cuelen** in 1610 to compute π to 35 decimal places. The number was engraved on his tombstone, and to this day in Germany, π is sometimes called "the Ludolphine number."

2. In 1760, **Comte de Buffon** (1707–1788) used probability and his famous needle method to determine π. What was his method?

3. Historically, a number of infinite series have been used to compute π. Use a calculator or a computer to check out these series to twenty places.

a. **Gottfried Leibniz** used $\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots)$.

b. **Leonhard Euler** used $\pi^2 = 6[(\frac{1}{1})^2 + (\frac{1}{2})^2 + (\frac{1}{3})^2 + \ldots]$.

4. a. Research the complete 1897 Indiana State House Bill Number 246. Find within it statements that contradict both one another and elementary geometry.

b. The bill makes the assumption that a circle and a square have equal areas if they have equal perimeters. Show that this is incorrect. The isoperimetric problem asks for the figure of largest area with fixed perimeter. Investigate the history of this problem.

RELATED READING

Beckmann, Petr. *A History of π*. New York: St. Martin's Press, 1976.

Castellanos, Dario. "The Ubiquitous π." *Mathematics Magazine* (Apr 1988) 67–98.

Davis, Philip, and Reuben Hersh. *The Mathematical Experience*. Boston: Birkhauser Boston, 1981.

Edgar, G.A. "Pi: Difficult or Easy?" *Mathematics Magazine* (June 1987) 141–150.

Edington, E. House Bill No. 246, Indiana State Legislature, 1897. *Proceedings of the Indiana Academy of Sciences* 45 (1935) 206–210.

Eves, Howard. *An Introduction to the History of Mathematics*. New York: Holt, Rinehart and Winston, 1990.

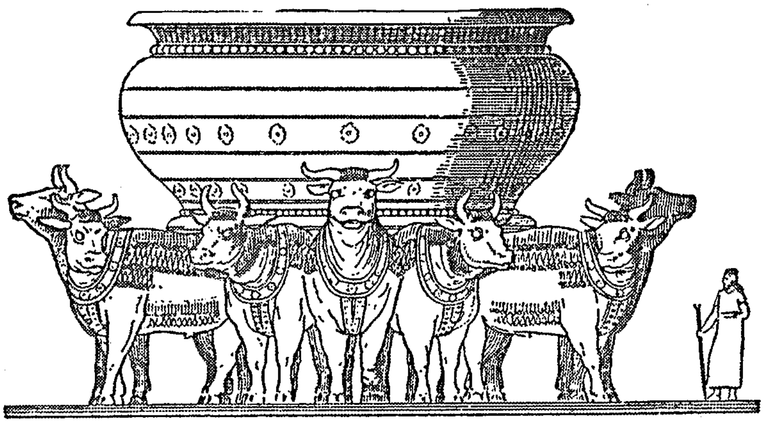
Lotspeich, Richard. "Archimedes' Pi—An Introduction to Iteration." *Mathematics Teacher* (Dec 1988) 208–210.

Peterson, Ivars. *Islands of Truth: A Mathematical Mystery Cruise*. New York: W.H. Freeman, 1990.

Shilgalis, Thomas W. "Archimedes and Pi." *Mathematics Teacher* (Mar 1989) 204–206.

ANCIENT REFERENCES TO PI

2



The molten sea.
From **A History of π,**
©1971 by Petr Beckmann,
St. Martin's Press, Inc.,
New York, NY.

The Greek letter pi (π) is used as the symbol for the ratio of the circumference of a circle to its diameter, which equals 3.14159 . . . . While many mathematics students probably take this ratio for granted, it wasn't always neatly provided in textbooks. On the contrary, the concept of pi has a long history of development and application.

By 1850 B.C., the ancient Egyptians had squared the circle to get $(4/3)^4$, or about 3.1605, as a value for π. According to the Bible, some 900 years later **Solomon** built a palace and a building complex, probably using the mathematics developed by the Egyptians to aid in its construction.

> *And he made the molten sea, ten cubits from one brim to the other: it was round all about, and its height was five cubits: and a line of thirty cubits did compass it round about.*
>
> —1 Kings 7:23 (A similar verse appears in 2 Chronicles 4:2.)

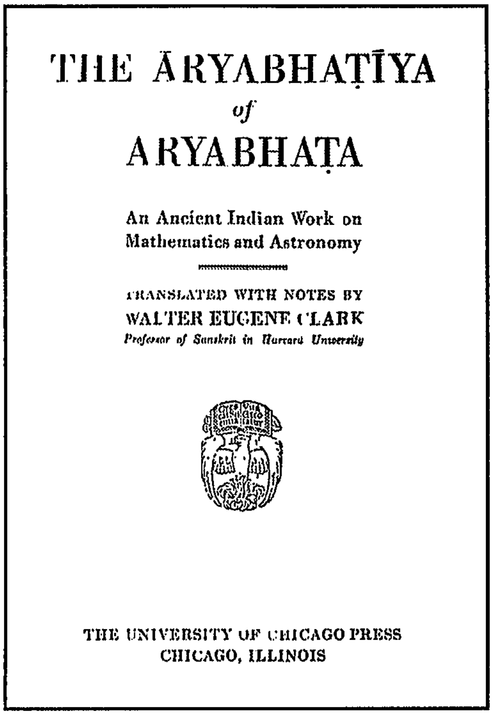
The cubit was a unit of measure representing the distance from a man's elbow to the end of his middle finger, about 17 to 22 inches. The molten sea was a high bowl or tank supported by 12 statues of oxen, in which priests washed in preparation for religious ceremonies.

> *Below its brim were ornamental buds encircling it all around, ten to a cubit, all the way around the sea. The ornamental buds were cast in two rows when it was cast. It stood on twelve oxen: three looking toward the north, three looking toward the west, three looking toward the south, and three looking toward the east; the sea was set upon them; and all their parts pointed inward. It was a hand-breadth thick; and its brim was shaped like the brim of a cup, like a lily blossom. It contained two thousand baths.*
>
> —1 Kings 7:24–26

In ancient Greece, **Archimedes** (ca 287–212 B.C.) found π to be between $223/71$ and $22/7$ by circumscribing and inscribing regular polygons about a circle. Six hundred years later, in a set of Indian manuscripts called *Siddhantas (Systems of Astronomy,* A.D. 400), the value for $\pi = 3^{177}/1250$, or 3.1416. It's thought that fifth-century Hindu mathematicians used Archimedes' method to find the value of π, but this isn't known for certain.

Chinese mathematicians, who had always used the decimal system, also searched for values of π. In A.D. 718, one Chinese document shows that $\pi = 92/29 = 3.1724 \ldots$. **Liu Hui** (ca A.D. 250) definitely used a variation of Archimedes' method, inscribing a polygon of 192 sides to find $3.141024 < \pi \leq 3.142704$. Taking it further, he found $\pi = 3.14159$ by inscribing a polygon of 3,072 sides.

For more on pi, see vignette 80. ★


THE ĀRYABHAṬĪYA

of

ĀRYABHAṬA

An Ancient Indian Work on
Mathematics and Astronomy

TRANSLATED WITH NOTES BY
WALTER EUGENE CLARK
Professor of Sanskrit in Harvard University

THE UNIVERSITY OF CHICAGO PRESS
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An important description of the Hindu numerical system, the* Aryabhatiya *is one of the earliest known publications to use algebra.

ACTIVITIES

1. Hindu mathematician **Aryabhata** (b A.D. 476) states in his manuscript *Aryabhatiya* (A.D. 499): "Add 4 to 100, multiply by 8, and add 62,000. The result is approximately the circumference of a circle on which the diameter is 20,000." What value of π does this situation yield?

2. The molten sea is described as "round all about," suggesting a circle. What is the length of the circumference of this circle? Of the diameter? Show that these dimensions yield the result $\pi = 3$.

3. A bath was a liquid measure equal to approximately six gallons. How many gallons of water could the molten sea contain? Use the given dimensions to show that priests would have needed ladders or some similar device to bathe in the molten sea. Do your discoveries seem reasonable to you?

4. The ancient Egyptians used the formula $(d - d/9)^2$ for the area of a circle with diameter d. What value of π does this formula yield?

5. Modern computers have been used to find π to thousands of decimal places. What algorithms do computers use to compute π?

RELATED READING

Beckmann, Petr. *A History of π.* New York: St. Martin's Press, 1976.

Dunham, William. *The Mathematical Universe: An Alphabetical Journey Through the Great Proofs, Problems, and Personalities.* New York: John Wiley, 1994.

Gillings, Richard J. *Mathematics in the Time of the Pharaohs.* Cambridge, MA: MIT Press, 1972.

Kasner, Edward, and James R. Newman. *Mathematics and the Imagination.* Redmond, WA: Tempus Books, 1989.

Pappas, Theoni. *More Joy of Mathematics.* San Carlos, CA: Wide World/Tetra, 1991.