

Momentum and Collisions

Problem G**ELASTIC COLLISIONS****PROBLEM**

American juggler Bruce Sarafian juggled 11 identical balls at one time in 1992. Each ball had a mass of 0.20 kg. Suppose two balls have an elastic head-on collision during the act. The first ball moves away from the collision with a velocity of 3.0 m/s to the right, and the second ball moves away with a velocity of 4.0 m/s to the left. If the first ball's velocity before the collision is 4.0 m/s to the left, what is the velocity of the second ball before the collision?

SOLUTION**1. DEFINE****Given:**

$$m_1 = m_2 = 0.20 \text{ kg}$$

$$\begin{aligned} \mathbf{v}_{1,i} &= \text{initial velocity of ball 1} = 4.0 \text{ m/s to the left} \\ &= -4.0 \text{ m/s to the right} \end{aligned}$$

$$\mathbf{v}_{1,f} = \text{final velocity of ball 1} = 3.0 \text{ m/s to the right}$$

$$\begin{aligned} \mathbf{v}_{2,f} &= \text{final velocity of ball 2} = 4.0 \text{ m/s to the left} \\ &= -4.0 \text{ m/s to the right} \end{aligned}$$

Unknown:

$$\mathbf{v}_{2,i} = \text{initial velocity of ball 2} = ?$$

2. PLAN Choose the equation(s) or situation: Use the equation for the conservation of momentum to determine the initial velocity of ball 2. Because both balls have identical masses, the mass terms cancel.

$$m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i} = m_1 \mathbf{v}_{1,f} + m_2 \mathbf{v}_{2,f}$$

$$\mathbf{v}_{1,i} + \mathbf{v}_{2,i} = \mathbf{v}_{1,f} + \mathbf{v}_{2,f}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\mathbf{v}_{2,i} = \mathbf{v}_{1,f} + \mathbf{v}_{2,f} - \mathbf{v}_{1,i}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\mathbf{v}_{2,i} = 3.0 \text{ m/s} - 4.0 \text{ m/s} - (-4.0 \text{ m/s})$$

$$\mathbf{v}_{2,i} = 3.0 \text{ m/s to the right}$$

4. EVALUATE

Confirm your answer by making sure that kinetic energy is also conserved.

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$v_{1,i}^2 + v_{2,i}^2 = v_{1,f}^2 + v_{2,f}^2$$

$$(-4.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2 = (3.0 \text{ m/s})^2 + (-4.0 \text{ m/s})^2$$

$$16 \text{ m}^2/\text{s}^2 + 9.0 \text{ m}^2/\text{s}^2 = 9.0 \text{ m}^2/\text{s}^2 + 16 \text{ m}^2/\text{s}^2$$

$$25 \text{ m}^2/\text{s}^2 = 25 \text{ m}^2/\text{s}^2$$

ADDITIONAL PRACTICE

- The moon's orbital speed around Earth is 3.680×10^3 km/h. Suppose the moon suffers a perfectly elastic collision with a comet whose mass is 50.0 percent that of the moon. (A partially inelastic collision would be a much more realistic event.) After the collision, the moon moves with a speed of -4.40×10^2 km/h, while the comet moves away from the moon at -5.740×10^3 km/h. What is the comet's speed before the collision?

3. The first astronaut to walk in outer space without being tethered to a spaceship was Capt. Bruce McCandless. In 1984, he used a jet backpack, which cost about \$15 million to design, to move freely about the exterior of the space shuttle *Challenger*. Imagine two astronauts working in outer space. Suppose they have equal masses and accidentally run into each other. The first astronaut moves 5.0 m/s to the right before the collision and 2.0 m/s to the left afterwards. If the second astronaut moves 5.0 m/s to the right after the perfectly elastic collision, what was the second astronaut's initial velocity?

5. Jana Novotna of what is now the Czech Republic has the strongest serve among her fellow tennis players. In 1993, she sent the ball flying with a speed of 185 km/h. Suppose a tennis ball moving to the right at this speed hits a moveable target of unknown mass. After the one-dimensional, perfectly elastic collision, the tennis ball bounces to the left with a speed of 80.0 km/h. If the tennis ball's mass is 5.70×10^{-2} kg, what is the target's mass? (Hint: Use the conservation of kinetic energy to solve for the second unknown quantity.)

7. A dump truck used in Canada has a mass of 5.50×10^5 kg when loaded and 2.30×10^5 kg when empty. Suppose two such trucks, one loaded and one empty, crash into each other at a monster truck show. The trucks are supplied with special bumpers that make a collision almost perfectly elastic. If the trucks hit each other at equal speeds of 5.00 m/s and the less massive truck recoils to the right with a speed of 9.10 m/s, what is the velocity of the full truck after the collision?