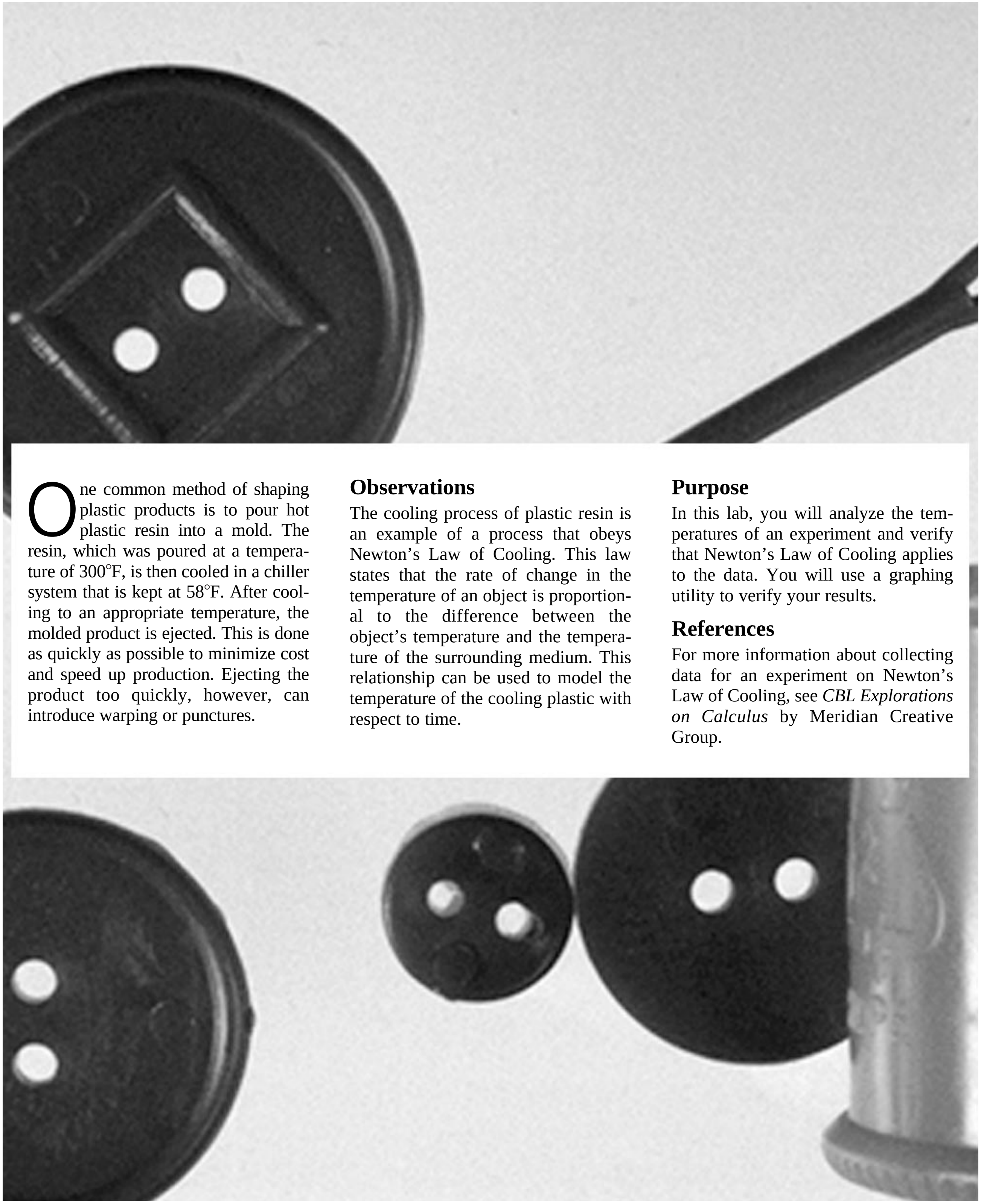
LAB 7

NEWTON'S LAW OF COOLING

Exponential Decay



One common method of shaping plastic products is to pour hot plastic resin into a mold. The resin, which was poured at a temperature of 300°F, is then cooled in a chiller system that is kept at 58°F. After cooling to an appropriate temperature, the molded product is ejected. This is done as quickly as possible to minimize cost and speed up production. Ejecting the product too quickly, however, can introduce warping or punctures.

Observations

The cooling process of plastic resin is an example of a process that obeys Newton's Law of Cooling. This law states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. This relationship can be used to model the temperature of the cooling plastic with respect to time.

Purpose

In this lab, you will analyze the temperatures of an experiment and verify that Newton's Law of Cooling applies to the data. You will use a graphing utility to verify your results.

References

For more information about collecting data for an experiment on Newton's Law of Cooling, see *CBL Explorations on Calculus* by Meridian Creative Group.

Data

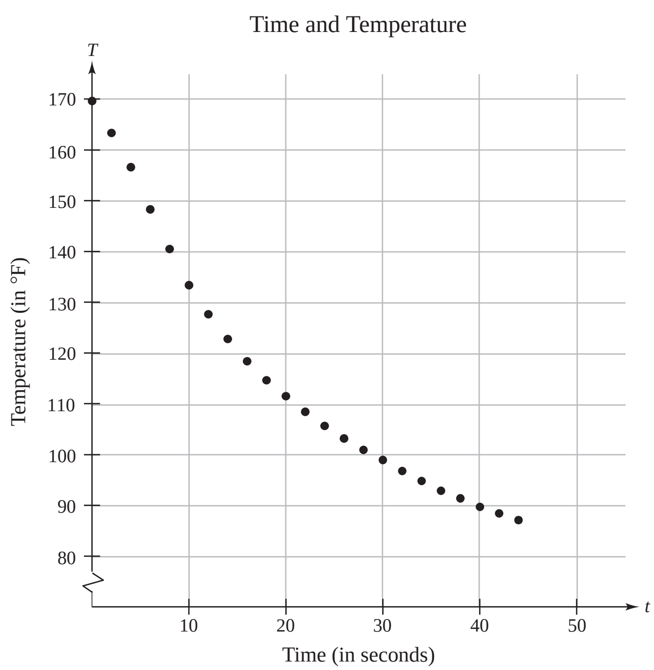
The temperature of a cup of water was taken at two-second intervals during a 44-second period and recorded in the table below. The room temperature was measured at 69.548°F, and the water temperature at time $t = 0$ was measured at 169.628°F. The time t is given in seconds and the temperature T is given in degrees Fahrenheit.

Time, *t*	0	2	4	6	8	10
Temperature, *T*	169.628	163.328	156.596	148.280	140.468	133.358

Time, *t*	12	14	16	18	20	22
Temperature, *T*	127.634	122.756	118.364	114.620	111.488	108.410

Time, *t*	24	26	28	30	32	34
Temperature, *T*	105.638	103.154	100.904	98.924	96.746	94.784

Time, *t*	36	38	40	42	44
Temperature, *T*	92.858	91.364	89.672	88.394	87.134



The data in the table and the scatter plot are stored in the graphing utility file called LAB07.

Exercises

Name ______________________________

Date ______________________ Class ____________

Instructor ______________________________

1. ***Describing What Happened.*** Use the scatter plot to describe the rate at which the water cooled. Do you think the temperature of the water ever reached 69.548°F? Why or why not?

2. ***Finding a General Solution.*** Newton's Law of Cooling can be represented mathematically by the differential equation

$$\frac{dy}{dt} = k(y - L)$$

where y is the temperature of the object at time t, k is a proportionality constant, and L is the temperature of the surrounding medium. Find the general solution of this differential equation. Describe in real-life terms what each unknown represents. Describe what you think the general equation's arbitrary constant represents.

3. ***Finding a Particular Solution.*** Apply the differential equation and its general solution from Exercise 2 to the temperatures of the water as described in this lab's Data. Show your work and record your results below. Describe in words what the general equation's arbitrary constant represents in the context of this experiment. Does this description confirm or contradict your description in Exercise 2? If it is a contradiction, rewrite the description.

Differential equation: $\frac{dy}{dt} =$

General solution: $y =$

Particular solution: $y =$

4. ***Will the Water Reach Room Temperature?*** Does the temperature of the water ever reach room temperature? Why or why not? Explain how you determined your conclusion. If the water does reach room temperature, find the time t when this occurs. Does this result support the conclusion you drew in Exercise 1?

5. ***First Derivative.*** Verbally describe the graph of the first derivative of the particular solution from Exercise 3. Graph the first derivative to verify your description. Explain what the value of the first derivative means at a time t.

6. ***Second Derivative.*** Verbally describe the graph of the second derivative of the particular solution from Exercise 3. Graph the second derivative to verify your description. Explain what the value of the second derivative means at a time t.

7. ***Comparing Solutions.*** A numerical solution of the differential equation

$$\frac{dy}{dt} = k(y - L)$$

with respect to the temperatures of the water that is described in this lab's Data can be found using a graphing utility. Compare the numerical solution the graphing utility found to the actual temperature data. Do you think the numerical solution found using the graphing utility is a good fit to the actual temperature data? Compare the numerical solution the graphing utility found to the particular solution you found in Exercise 3. If the solutions produce different approximations of the temperature data, explain why.

8. ***Cooling Plastic.*** Apply the differential equation and its general solution from Exercise 2 to the cooling plastic discussed in the Introduction of this lab. Is there enough information to determine the value of the proportionality constant k? If so, find its value. If not, explain what additional information is needed to determine the value of k.

Differential equation: $\frac{dy}{dt} =$

General solution: $y =$

Particular solution: $y =$

9. ***Is Cooler Better?*** If speed of production is important, how could you make the cooling process go faster? Do you think this is a viable solution? Why or why not?