

While designs for rotary engines date back as far as the 1500's, the Wankel rotary engine was developed by Felix Wankel in the 1950's. The design of the rotary engine uses some interesting geometric shapes. The rotor in a Wankel engine is shaped like a slightly bulged equilateral triangle and the shape of the rotor's housing chamber is an epitrochoid.

### Observations

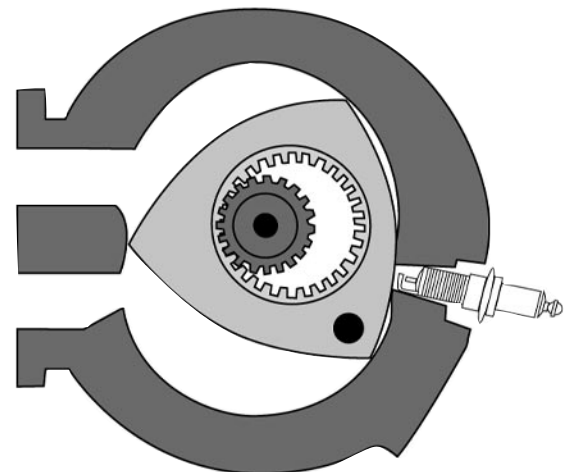
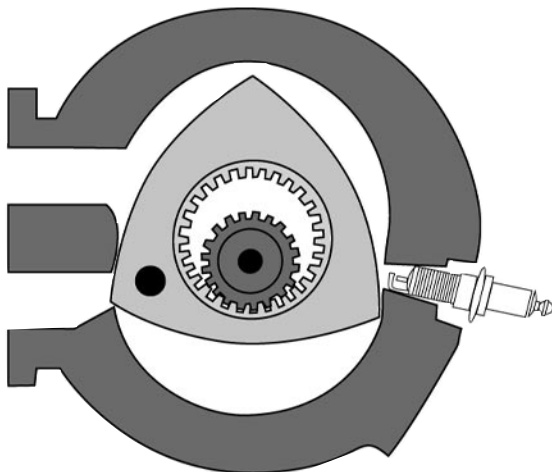
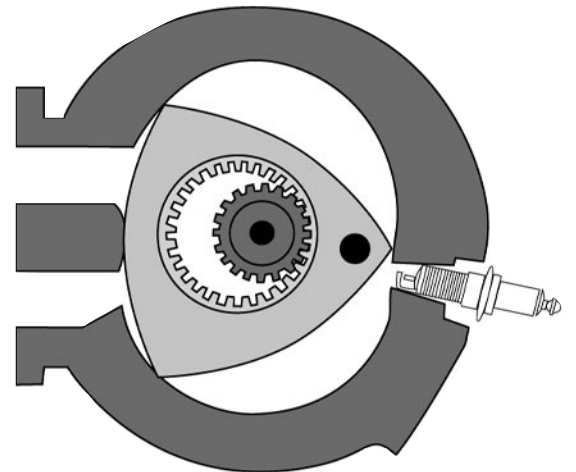
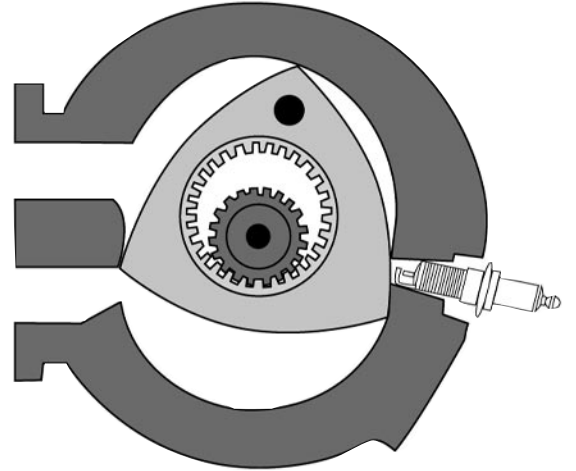
The Wankel engine has some advantages over the common piston engine. The Wankel engine has fewer moving parts. For example, there are no valves, connecting rods, or timing belts. Thus, the Wankel engine should have less labor and material costs. However, the Wankel engine is less thermodynamically efficient and uses more fuel than most piston engines.

### Purpose

In this lab, you will analyze the housing chamber. You will use numerical methods and use a graphing utility to study the measurements of a chamber design.

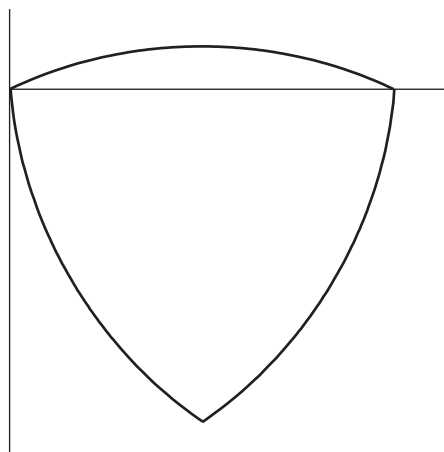
### References

For more information about the Wankel rotary engine, see *The Wankel Engine: Design, Development, Application* by Jan D. Norbye.

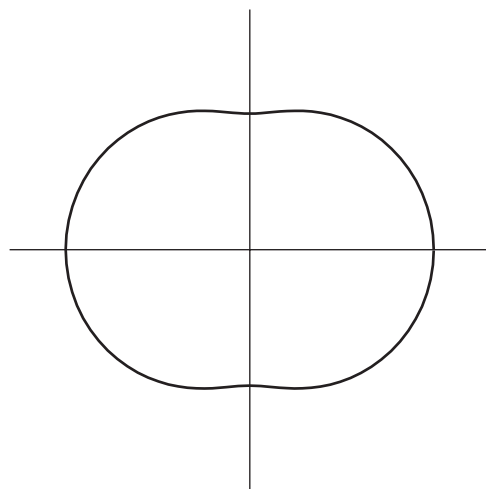


## Data

The rotor in a Wankel engine is shaped like a slightly bulged equilateral triangle, as shown in the graph below.




The rotor's housing chamber has the shape of an epitrochoid. An epitrochoid is a curve traced by a point on a circle rolling around another circle without slipping. An example of an epitrochoid is shown below.



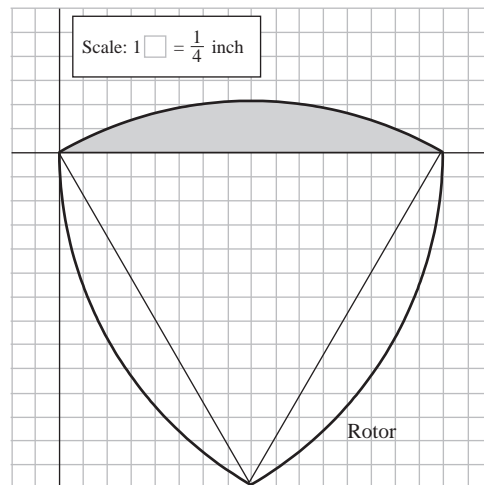


2. **A Reasonable Estimate?** Inscribe and circumscribe an equilateral triangle on the rotor in the figure in Exercise 1. Then find the areas of the triangles. Use these areas to determine if your estimate in Exercise 1 is reasonable.

-  3. **Using Integration and Geometry to Find Area.** An equilateral triangle has been inscribed in the rotor as shown in the figure. The area of the shaded region is given by

$$\int_0^4 (-2\sqrt{3} + \sqrt{16 - (x - 2)^2}) dx.$$

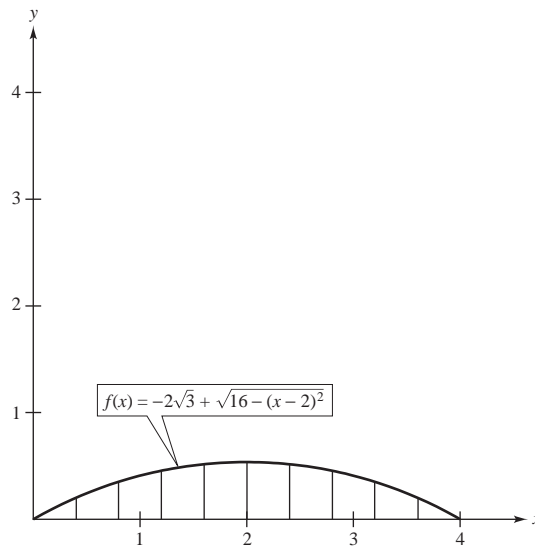
Use a graphing utility to evaluate this integral. Then use the result of the integration and the formula for the area of a triangle to find the total area of the rotor. According to Exercise 2, is this result reasonable?



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**4. Using a Numerical Method.** Several measurements of a region of a rotor are given in the table. Use the measurements, the Trapezoidal Rule and a graphing utility to estimate the area of the region where  $x$  and  $y$  are measured in inches as shown in the figure. Determine the possible error of this estimate. Do you think this is an acceptable amount of error? Why or why not? How can the error in the estimate of the region's area be reduced?



$x$	0.00	0.40	0.80	1.20	1.60	2.00	2.40
$y$	0.00	0.20	0.35	0.45	0.52	0.54	0.52

$x$	2.80	3.20	3.60	4.00
$y$	0.45	0.35	0.20	0.00

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**5. Another Numerical Method.** Repeat Exercise 4 using Simpson's Rule and a graphing utility. Which method gives a better estimate? If the number of measurements is increased, which method do you think will be better: the Trapezoidal Rule or Simpson's Rule? Explain your reasoning.

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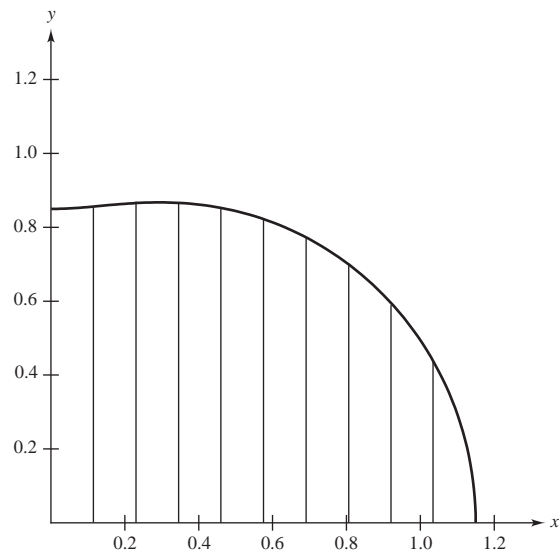


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**6. Estimating the Area of a Housing Chamber.** The measurements of a region of a housing chamber are given in the table. Choose one of the numerical methods used in this lab to estimate the area of the chamber where  $x$  and  $y$  are measured in inches as shown in the figure. Then modify the graphing utility commands given for the earlier methods and calculate an area estimate. Explain how you could use your estimate to approximate the area of the entire housing chamber.

$x$	0.000	0.115	0.230	0.345	0.460	0.575	0.690
$y$	0.850	0.848	0.854	0.848	0.835	0.804	0.748

$x$	0.805	0.920	1.035	1.150
$y$	0.680	0.574	0.425	0.000




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