LAB 5

PACKAGING

Optimization

There are many decisions to be made when designing a package for a product. For example, what will be the shape of the package? How much will the package need to hold? What will be the dimensions of the package? With these questions, it is easy to see that the packaging of some products can be more complicated than the product itself.

Observations

Making packages is a big industry. There are about 500 billion packages used every year in the United States. More than half of these packages contain foods or beverages. Of the 500 billion packages used every year, over 100 billion are cans. It takes 10 million tons of steel to make these cans.

Purpose

In this lab, you will analyze different package shapes and their optimal sizes. You will use the first and second derivative and a graphing utility to study the different package shapes.

References

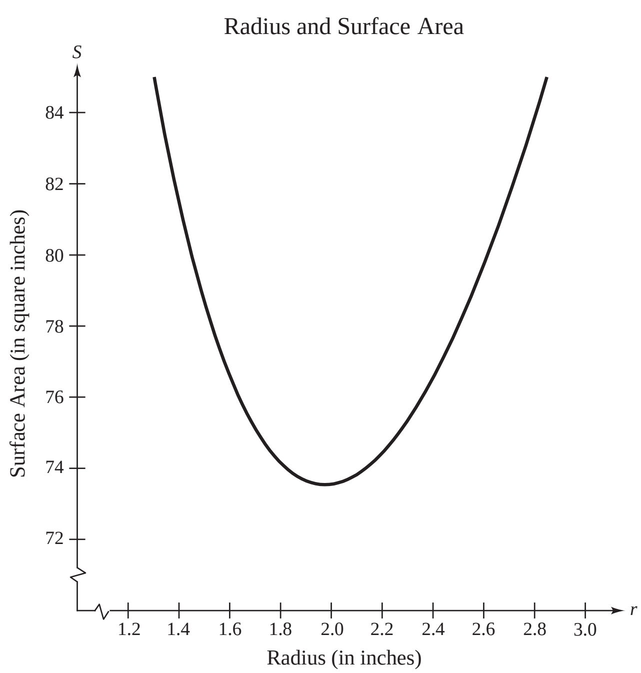
For more information about packaging, see the *Handbook of Package Engineering* from Technomic Publishing.

Data

The table below gives the approximate measurements in inches of several common items packed in cylindrical containers.

Product	Radius (in.)	Height (in.)	Volume (in.^{3})
Baking powder	1.25	3.65	17.92
Cleanser	1.45	7.50	49.54
Coffee	1.95	5.20	62.12
Coffee creamer	1.50	6.85	48.42
Frosting	1.63	3.60	30.05
Pineapple juice	2.10	6.70	92.82
Soup	1.30	3.80	20.18
Tomato puree	1.95	4.40	52.56

An infinite number of dimensions can be used to construct a right circular container of a given volume. The graph below shows the relationship between the radius and surface area for containers that have a volume of 48.42 cubic inches.



A graph of the parametric equations is stored in the graphing utility file called LAB05.

Exercises

Name ______________________________

Date ____________________ Class ____________

Instructor ______________________________

1. ***Complete the Table.*** Use the equation for the surface area of a right circular cylinder and the dimensions given in this lab's Data to complete the table.

Surface Area of a Right Circular Cylinder: $S = 2\pi r^2 + 2\pi rh$

Product	**Surface Area (in.^{2})**
Baking powder	
Cleanser	
Coffee	
Coffee creamer	
Frosting	
Pineapple juice	
Soup	
Tomato puree	

2. ***The Only One?*** The baking powder container described in this lab's Data has a radius of 1.25 inches, a surface area of 38.48 square inches, and a volume of 17.92 cubic inches. Is it possible to design another baking powder container with the same surface area and volume but a different radius? Why or why not? If it is possible, find the dimensions of another container having a surface area of 38.48 square inches and a volume of 17.92 cubic inches

____

____

____

____

____

____

3. ***Think About It.*** Do you think each of the products listed in Exercise 1 uses a container that minimizes surface area for the volume given in this lab's Data? Why or why not?

4. ***Designing a Container With a Specified Volume.*** Suppose you are designing a baking powder container that has a volume of 17.92 cubic inches. Use the equations for the surface area of a cylinder and the volume of a cylinder to develop an equation relating the surface area S and the radius r. (Note: the volume of a right circular cylinder is $V = \pi r^2 h$.)

   Use a graphing utility to plot the equation. Determine the radius of the baking powder container that minimizes surface area. Is this radius larger than, smaller than, or equal to the radius given for the baking powder container in this lab's Data? Does the baking powder container given in this lab's Data minimize surface area?

5. ***Finding the "Optimal" Surface Area.*** Repeat Exercise 4 for each of the products listed below and record the "optimal" radius in the table. Then use the volume and radius to determine the height and surface area for each container.

Product	**Volume (in.^{3})**	**Radius (in.)**	**Height (in.)**	**Surface Area (in.^{2})**
Cleanser	49.54			
Coffee	62.12			
Coffee creamer	48.42			
Frosting	30.05			
Pineapple juice	92.82			
Soup	20.18			
Tomato puree	52.56			

6. ***A Different Container Shape.*** Design a rectangular container with a square base for each of the following volumes and heights. The volume of a rectangular container with a square base is

$$V = x^2h$$

and the surface area is

$$S = x^2 + 4xh.$$

Volume (in.^{3})	**Height (in.)**	**Side of Base (in.)**	**Surface Area (in.^{2})**
17.92	3.65		
49.54	7.50		
62.12	5.20		
48.42	6.85		
30.05	3.60		
92.82	6.70		
20.18	3.80		
52.56	4.40		

Compare the rectangular containers with square bases to the cylinders that have the same volume in Exercise 1. What advantages do the rectangular containers have over the cylinders? What disadvantages do they have?

7. ***Optimizing a Rectangular Container.*** In Exercise 6, did you design each rectangular container such that surface area was minimized? For a fixed volume V, how would you determine the minimum surface area of a rectangular container with a square base? For a fixed surface area S, how would you determine the maximum volume of a rectangular container with a square base?

8. ***Why not?*** Determine whether the surface area you found in Exercise 1 for each container is equal to the "optimal" surface area you found for each container in Exercise 5. Give several reasons why a company would make a container that does not use an "optimal" surface area.