LAB 3

Falling Objects

First and Second Derivatives

To study the motion of an object under the influence of gravity, we need equipment to track the motion of the object. We can use calculus to analyze the data. Calculus can be used to determine the object's position, velocity, and acceleration due to gravity.

Observations

In theory, the position of a free-falling object (neglecting air resistance) is given by

$$s(t) = \frac{1}{2}gt^2 + v_0^{t} + s_0,$$

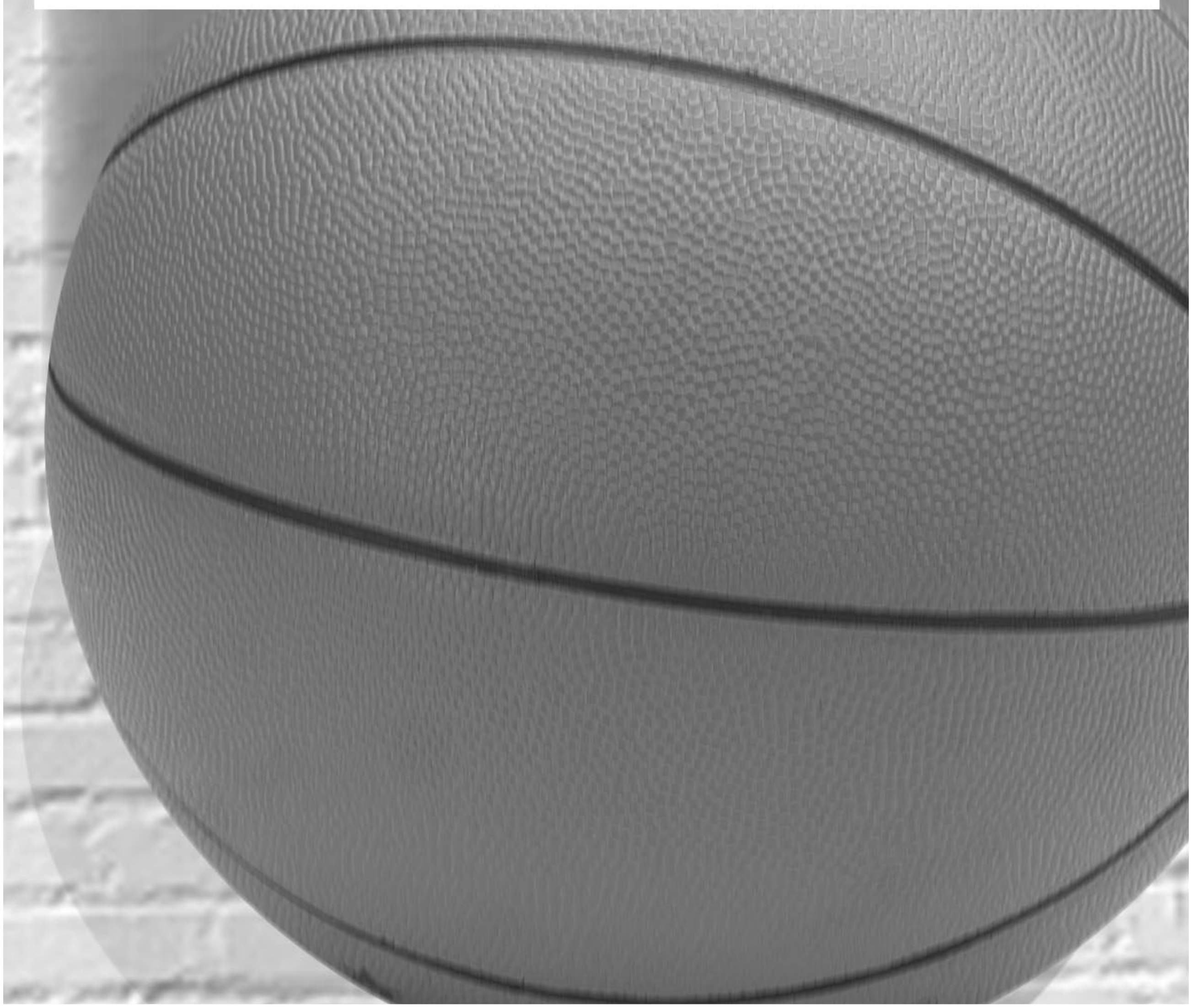
where g is the acceleration due to gravity, t is the time, v_0 is the initial velocity, and s_0 is the initial height.

Purpose

In this lab, you will analyze the data of a free-falling object collected during an experiment. You will use the first and second derivative and a graphing utility to study the motion of the object.

References

For more information about experiments involving the study of motion of an object, see *CBL Explorations in Calculus* by Meridian Creative Group.

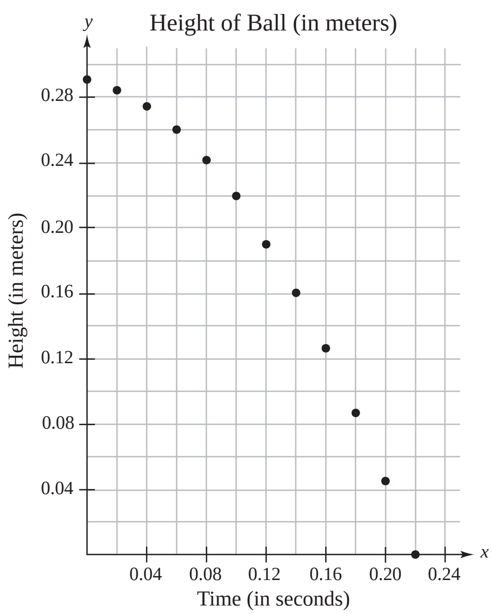


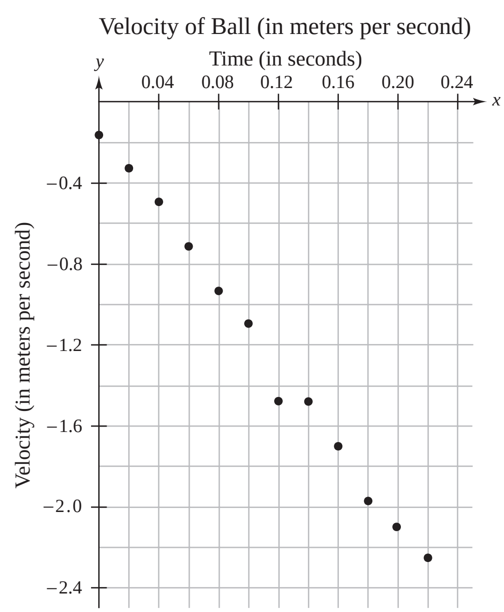
Data

The positions of a falling ball at time intervals of 0.02 second are given in the table below.

Time (sec)	Height (meters)	Velocity (meters/sec)
0.00	0.290864	−0.16405
0.02	0.284279	−0.32857
0.04	0.274400	−0.49403
0.06	0.260131	−0.71322
0.08	0.241472	−0.93309
0.10	0.219520	−1.09409
0.12	0.189885	−1.47655
0.14	0.160250	−1.47891
0.16	0.126224	−1.69994
0.18	0.086711	−1.96997
0.20	0.045002	−2.07747
0.22	0.000000	−2.25010

Scatter plots of the data are given below.





A graph of the parametric equations is stored in the graphing utility file called LAB03.

Exercises

Name ______________________________

Date ______________________ Class ______________

Instructor ______________________________

1. ***What Type of Model?*** What type of model seems to be the best fit for the scatter plot of the heights of the falling ball? What type of model seems to be the best fit for the scatter plot of the velocity of the falling ball? Describe any relationships you see between the two models.

2. ***Modeling the Position Function.*** A model of a position function takes the form

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0.$$

Use a graphing utility to find the position function of the falling ball described in this lab's Data and record the result below. Use this position function to determine the ball's initial height, initial velocity, velocity function, and acceleration function and record the results below.

Position Function: $s(t) =$

Initial Height: $s_0 =$

Initial Velocity: $v_0 =$

Velocity Function: $v(t) = s'(t) =$

Acceleration Function: $a(t) = s''(t) =$

3. ***A Good Fit?*** Is the velocity function you found in Exercise 2 a good fit? Why or why not? Use a graphing utility to analyze the velocity function graphically and numerically.

4. ***Modeling the Velocity Function.*** A model of a velocity function takes the form

$$v(t) = gt + v_0.$$

Use a graphing utility to find the velocity function of the falling ball described in this lab's Data and record the result below. Use this velocity function to determine the acceleration function and record the result below.

Velocity Function: $v(t) =$

Acceleration Function: $a(t) = v'(t) =$

5. ***What's the Difference?*** Of the velocity functions you found in Exercises 2 and 4, which one is a better fit to the data? Use a graphing utility to analyze the velocity functions graphically and numerically. Explain why these velocity functions which describe the same data are different.

6. ***Estimating Earth's Gravity.*** Of the acceleration functions you found in Exercises 2 and 4, which one is closer to the actual value of earth's gravity? (Note: The value of earth's gravity is approximately -9.8 m/sec^2.) Calculate the percent error of the closest estimate of earth's gravity. Do you think this is a good estimate? Why or why not?

Another Ball. For Exercises 7–9, use the following data.

Time (sec)	Height (meters)	Time (sec)	Height (meters)
0.00	0.806736	0.32	1.149180
0.02	0.857225	0.34	1.141500
0.04	0.904422	0.36	1.126130
0.06	0.946131	0.38	1.105280
0.08	0.985644	0.40	1.082230
0.10	1.020760	0.42	1.056980
0.12	1.052590	0.44	1.026250
0.14	1.080030	0.46	0.992230
0.16	1.103080	0.48	0.954912
0.18	1.122840	0.50	0.913203
0.20	1.137110	0.52	0.868201
0.22	1.149180	0.54	0.819907
0.24	1.156870	0.56	0.767222
0.26	1.160160	0.58	0.711244
0.28	1.161260	0.60	0.651974
0.30	1.156870	0.62	0.589411

7. Use a graphing utility to find a model for the position function of the data and record the result below. Use this position function to determine the ball's initial height and initial velocity. Do you think this ball was dropped or thrown? Explain your reasoning.

Position Function: $s(t) =$

Initial Height: $s_0 =$

Initial Velocity: $v_0 =$

8. Use a graphing utility to analyze the graph of the position function s from Exercise 7 and its derivative s'. For which values of the time t is s' positive? For which values of the time t is s' negative? What does the graph of s' tell you about the graph of s?

9. Use a graphing utility to analyze the tangent line to the graph of the position function s from Exercise 7 at different times. At what time is the slope of the tangent line horizontal? What is the velocity of the ball at this time?