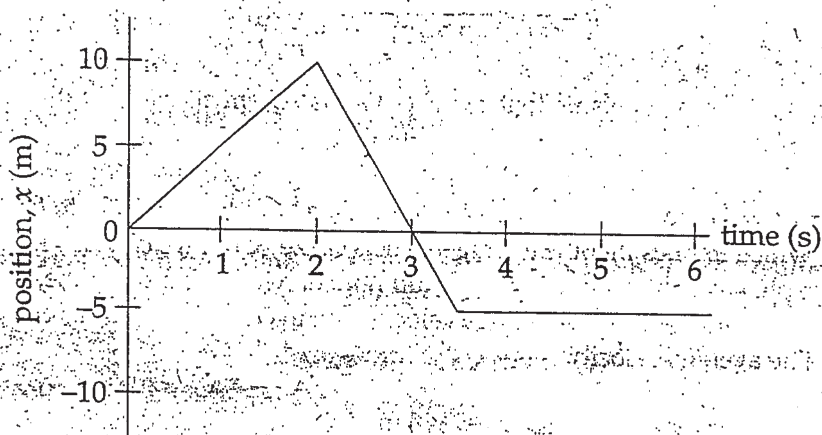
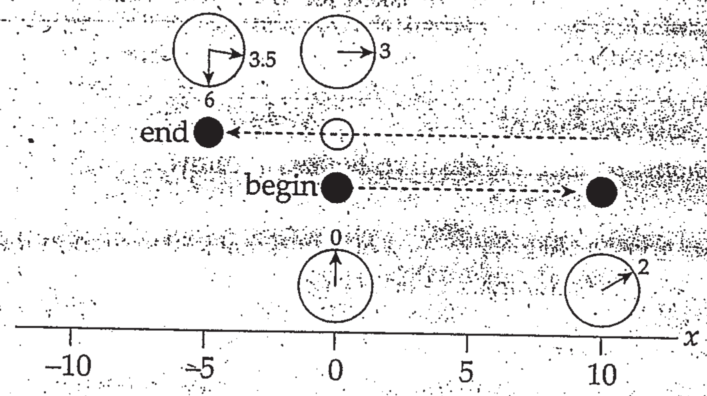
KINEMATICS WITH GRAPHS

So far, we have dealt with kinematics problems algebraically, but you should also be able to handle kinematics questions in which information is given graphically. The two most popular graphs in kinematics are position-vs.-time graphs and velocity-vs.-time graphs. For example, consider an object that's moving along an axis in such a way that its position x as a function of time t is given by the following position-vs.-time graph:



What does this graph tell us? It says that at time $t = 0$, the object was at position $x = 0$. Then, in the next two seconds, its position changed from $x = 0$ to $x = 10$ m.



Then, at time $t = 2$ s, it reversed direction and headed back toward its starting point, reaching $x = 0$ at time $t = 3$ s, and continued, reaching position $x = -5$ m at time $t = 3.5$ s. Then the object remained at this position, $x = -5$ m, at least through time $t = 6$ s. Notice how economically the graph embodies all this information!

We can also determine the object's average velocity (and average speed) during particular time intervals. For example, its average velocity from time $t = 0$ to time $t = 2$ s is equal to the distance traveled, $10 - 0 = 10$ m, divided by the elapsed time, 2 s:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{(10-0)\text{ m}}{(2-0)\text{ s}} = 5\text{ m/s}$$

Notice, however, that the ratio that defines the average velocity, $\Delta x / \Delta t$, also defines the slope of the x vs. t graph. Therefore, we know the following important fact:

The slope of a position-vs.-time graph gives the velocity.

What was the average velocity from time $t = 2$ s to time $t = 3.5$ s? The slope of the line segment joining the point $(t, x) = (2\text{ s}, 10\text{ m})$ to the point $(t, x) = (3.5\text{ s}, -5\text{ m})$ is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{(-5-10)\text{ m}}{(3.5-2)\text{ s}} = -10\text{ m/s}$$

The fact that $\bar{v}$ is negative tells us that the object's displacement was negative during this time interval; that is, it moved in the negative x direction. The fact that $\bar{v}$ is negative agrees with the observation that the slope of a line that falls to the right is negative. What is the object's average velocity from time $t = 3.5$ s to time $t = 6$ s? Since the line segment from $t = 3.5$ s to $t = 6$ s is horizontal, its slope is zero, which implies that the average velocity is zero, but we can also figure this out from looking at the graph, since the object's position did not change during that time.

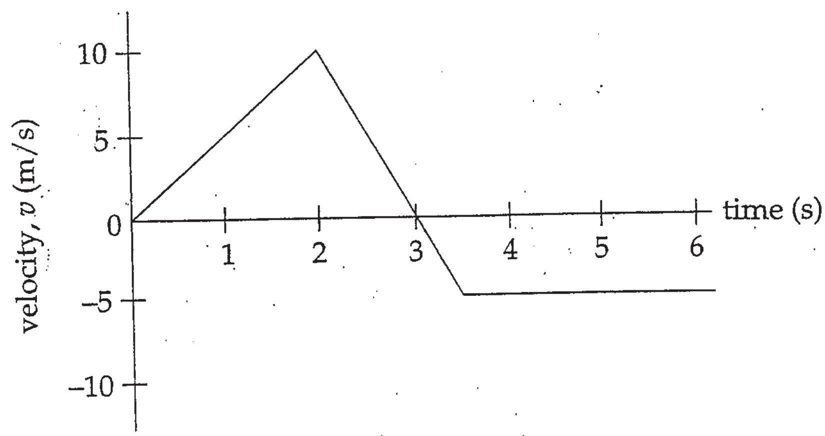
Finally, let's figure out the object's average velocity and average speed for its entire journey (from $t = 0$ to $t = 6$ s). The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{(-5-0)\text{ m}}{(6-0)\text{ s}} = -0.83\text{ m/s}$$

This is the slope of the imagined line segment that joins the point $(t, x) = (0 \text{ s}, 0 \text{ m})$ to the point $(t, x) = (6 \text{ s}, -5 \text{ m})$. The average speed is the total distance traveled by the object divided by the elapsed time. In this case, notice that the object traveled 10 m in the first 2 s, then 15 m (albeit backwards) in the next 1.5 s; it covered no additional distance from $t = 3.5$ s to $t = 6$ s. Therefore, the total distance traveled by the object is $d = 10 + 15 = 25$ m, which took 6 s, so

$$\text{average speed} = \frac{d}{\Delta t} = \frac{25 \text{ m}}{6 \text{ s}} = 4.2 \text{ m/s}$$

Let's next consider an object moving along a straight axis in such a way that its velocity, v, as a function of time, t, is given by the following velocity-vs.-time graph:



What does this graph tell us? It says that, at time $t = 0$, the object's velocity was $v = 0$. Over the first two seconds, its velocity increased steadily to 10 m/s. At time $t = 2$ s, the velocity then began to decrease (eventually becoming $v = 0$, at time $t = 3$ s). The velocity then became negative after $t = 3$ s, reaching $v = -5$ m/s at time $t = 3.5$ s. From $t = 3.5$ s on, the velocity remained a steady -5 m/s.

What can we ask about this motion? First, the fact that the velocity changed from $t = 0$ to $t = 2$ s tells us that the object accelerated. The acceleration during this time was

$$a = \frac{\Delta v}{\Delta t} = \frac{(10-0) \text{ m/s}}{(2-0) \text{ s}} = 5 \text{ m/s}^2$$

Notice, however, that the ratio that defines the acceleration, $\Delta v / \Delta t$, also defines the slope of the v vs. t graph. Therefore,

The slope of a velocity-vs.-time graph gives the acceleration.

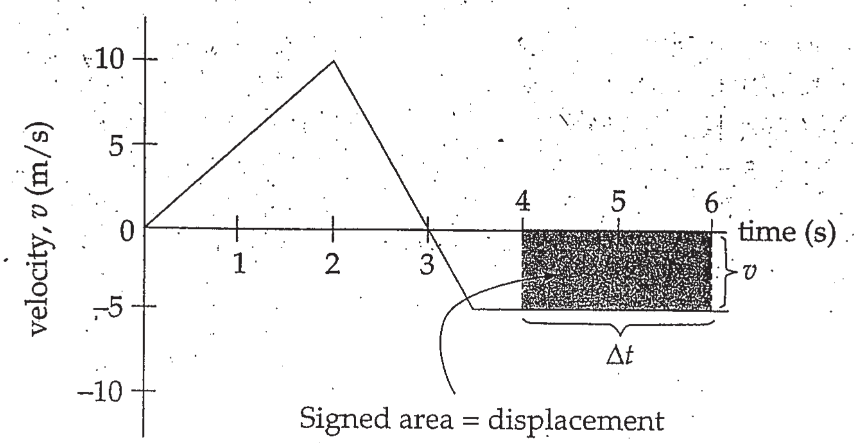
What was the acceleration from time $t = 2$ s to time $t = 3.5$ s? The slope of the line segment joining the point $(t, v) = (2 \text{ s}, 10 \text{ m/s})$ to the point $(t, v) = (3.5 \text{ s}, -5 \text{ m/s})$ is

$$a = \frac{\Delta v}{\Delta t} = \frac{(-5-10) \text{ m/s}}{(3.5-2) \text{ s}} = -10 \text{ m/s}^2$$

The fact that a is negative tells us that the object's velocity change was negative during this time interval; that is, the object accelerated in the negative direction. In fact, after time $t = 3$ s, the velocity became more negative, indicating that the direction of motion was negative at increasing speed. What is the object's acceleration from time $t = 3.5$ s to time $t = 6$ s? Since the line segment from $t = 3.5$ s to $t = 6$ s is horizontal, its slope is zero, which implies that the acceleration is zero, but you can also see this from looking at the graph; the object's velocity did not change during this time interval.

Another question can be asked when a velocity-vs.-time graph is given: How far did the object travel during a particular time interval? For example, let's figure out the displacement of the object from time $t = 4$ s to time $t = 6$ s. During this time interval, the velocity was a constant −5 m/s, so the displacement was $\Delta x = v\Delta t = (-5\text{ m/s})(2\text{ s}) = -10\text{ m}$.

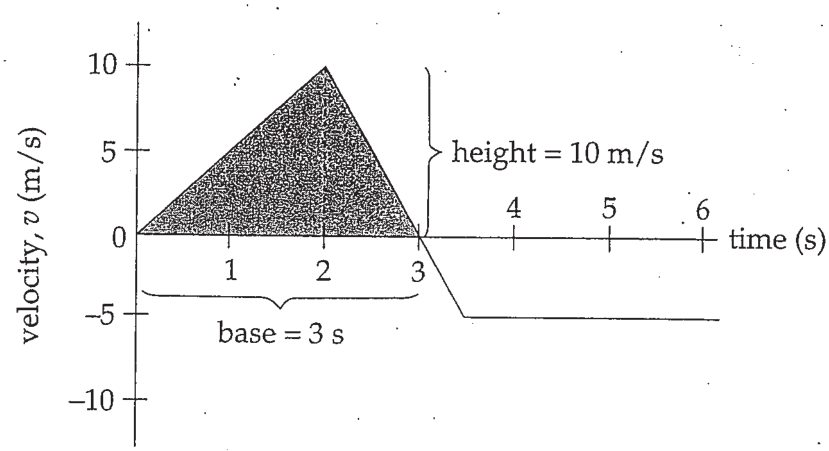
Geometrically, we've determined the area between the graph and the horizontal axis. After all, the area of a rectangle is *base* × *height* and, for the shaded rectangle shown below, the *base* is Δt, and the *height* is v. So, *base* × *height* equals $\Delta t \times v$, which is displacement.



We say *signed area* because regions below the horizontal axis are negative quantities (since the object's velocity is negative, its displacement is negative). Therefore, counting areas above the horizontal axis as positive and areas below the horizontal axis as negative, we can make the following claim:

Given a velocity-vs-time graph, the area between the graph and the t axis equals the object's displacement.

What is the object's displacement from time $t = 0$ to $t = 3$ s? Using the fact that displacement is the area bounded by the velocity graph, we figure out the area of the triangle shown below:



Since the area of a triangle is (1/2) × base × height, we find that $\Delta x = \frac{1}{2}(3\text{ s})(10\text{ m/s}) = 15\text{ m}$.

CHAPTER 2 REVIEW QUESTIONS

SECTION I: MULTIPLE CHOICE

1. An object that's moving with constant speed travels once around a circular path. Which of the following is/are true concerning this motion?

   I. The displacement is zero.
   II. The average speed is zero.
   III. The acceleration is zero.

   (A) I only
   (B) I and II only
   (C) I and III only
   (D) III only
   (E) II and III only

2. A rock is dropped off a cliff and strikes the ground with an impact velocity of 30 m/s. How high was the cliff?

   (A) 15 m
   (B) 20 m
   (C) 30 m
   (D) 45 m
   (E) 60 m

3. Which of the following is/are true?

   I. If an object's acceleration is constant, then it must move in a straight line.
   II. If an object's acceleration is zero, then its speed must remain constant.
   III. If an object's speed remains constant, then its acceleration must be zero.

   (A) I and II only
   (B) I and III only
   (C) II only
   (D) III only
   (E) II and III only

4. A baseball is thrown straight upward. What is the ball's acceleration at its highest point?

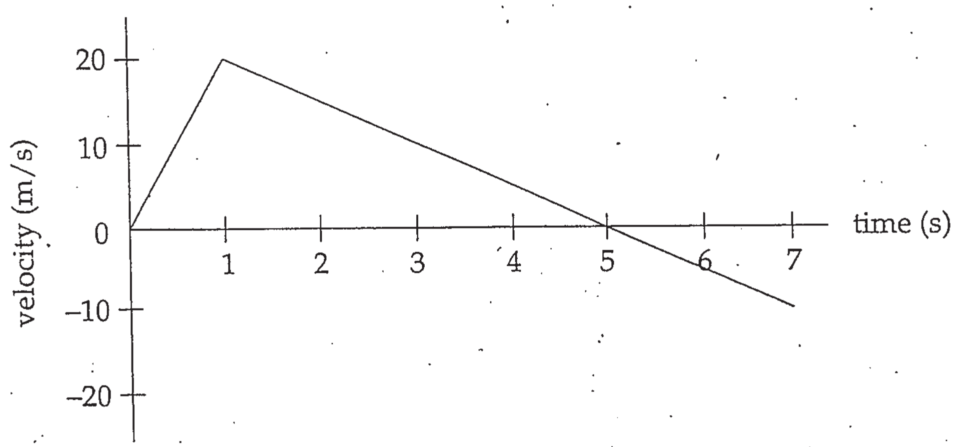
   (A) 0
   (B) $\frac{1}{2}g$, downward
   (C) g, downward
   (D) $\frac{1}{2}g$, upward
   (E) g, upward

5. How long would it take a car, starting from rest and accelerating uniformly in a straight line at 5 m/s^2, to cover a distance of 200 m?

   (A) 9.0 s
   (B) 10.5 s
   (C) 12.0 s
   (D) 15.5 s
   (E) 20.0 s

SECTION II: FREE RESPONSE

1. This question concerns the motion of a car on a straight track; the car's velocity as a function of time is plotted below.



(a) Describe what happened to the car at time $t = 1$ s.

(b) How does the car's average velocity between time $t = 0$ and $t = 1$ s compare to its average velocity between times $t = 1$ s and $t = 5$ s?

(c) What is the displacement of the car from time $t = 0$ to time $t = 7$ s?

(d) Plot the car's acceleration during this interval as a function of time.

(e) Plot the object's position during this interval as a function of time. Assume that the car begins at $s = 0$.